The Philosophy of Greek Mathematics and its influence on the Development of Arabic Civilization and Renaissance*

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Speaking about classical studies, we usually mean the study of ancient Greek and Latin literature, history, and philosophy that has come to us through the magnificent works of distinguished ancient writers. But a characteristic feature of Greek Philosophy was the consideration of the world and the ideas as one entity that cannot be studied separately. Thus, the Greeks developed Philosophy parallel to Mathematics, since the leading philosophers as Thales, Pythagoras, Plato, Aristotle were pioneers in the development of Mathematics and Physics, as a necessary means for revealing the secrets of Nature and the world around us.

These philosophers had traveled to Egypt and Babylon and were acquainted with the knowledge of the civilization thereof. They did not confine themselves to obtaining the results, but they searched for proofs and further development of the received ideas and knowledge. Greeks spread from the Italian peninsula to Asia Minor, formulated mathematics as a theoretical discipline, built on abstract notions rather than sensible objects, using formal demonstrations to prove the truth of their theorems. As Plutarchus preserves in his Convivial Questions[1], *"Plato censured Eudoxus and Archytas and Menaechmus for endeavouring to solve the doubling of the cube by instruments and mechanical constructions, thus trying by irrational means to find two mean proportionals: for in this way what is good in geometry would be corrupted and destroyed, falling back again into sensible objects and not rising upwards and laying hold of immaterial and eternal images, among which God has his being and remains for ever God."*

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[1] Plutarchus, *Quaestiones convivales*, Stephanus p. 718 : E7-F4 (Thesaurus Linguae Graecae)

Although Plato argued for the exclusive use of ruler and compass for the constructions in Geometry, Greeks trying to solve problems as the duplication of the cube, the trisection of an arbitrary angle and the quadrature of a circle and finding no solution in agreement with Plato's restrictions, they invented new methods as that of verging (νεύσις) and new curves as the conic sections, the conchoidal lines, the quadratrix etc[2]. Only in 19th century it was proved, using higher algebra that the above problems were impossible to be solved by the exclusive use of ruler and compass. Greek mathematicians manifested by their attitude the freedom of their thought; they had to reach their aim by any means, and they did it through the flourishing of a diversity of methods.

In the Timaeus, Plato tried to build up a complete system of physics, partly employing Pythagorean ideas. The Pythagoreans (5th century BC) were responsible for one of the first Greek astronomical theories. Believing that the order of the cosmos is fundamentally mathematical, they held that it is possible to discover the harmonies of the universe by contemplating the regular motions of the heavens. They constructed a model of cosmos postulating a central fire, about which the fixed stars, the five planets, the Sun, the Moon, the Earth and the Anti-Earth (Αντίχθων) revolve[3]. Subsequent Greek astronomy derived its character from a comment ascribed to Plato, in the 4th century BC, who is reported to have instructed the astronomers to "save the phenomena" in terms of uniform circular motion.[4]

In his Physica and Mechanica, Aristotle put the foundations of Physics; his theory held for almost two millennia, until Isaac Newton and Galileo refounded Physics on the base of experiment and mathematical expression of natural laws. Aristotle put also the logical base on which the Greek Mathematics was developed. In his Posterior Analytica,[5] he clarified the importance of building the science on first principles (elements whose

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[2] Tomas, Ivor, *Greek Mathematical Works*, in 2 vols, Chapter IX, Special Problems. Loeb Classical Library, London 1991 (first edition 1939).

[3] Alexander, *In Aristotelis metaphysica commentaria*, p.37.18-39.3 (Thesaurus Linguae Graecae)

[4] Ancient *Middle Eastern and Greek Astronomy*, Britannica CD. Version 97. Encyclopaedia Britannica, Inc., 1997

[5] Aristoteles, *Analytica Posteriora*, I. Bekker 76b 3-22 (Thesaurus Linguae Graecae)

existence cannot be proved), some of them peculiar to each science and some common, definitions, and postulates (propositions assumed and used without demonstration). He also stressed the dealing with abstract notions rather than figures: *"The geometer draws no conclusion from the existence of the particular line of which he speaks, but from what his diagrams represent"*[6]. In Alexandria of Egypt, the center of Hellenistic science, Euclid collected and systematized the accumulated knowledge of Greeks according to the Platonic and Aristotelian ideas. Characteristic example is his "Elements" that included the most important results obtained from the previous mathematicians. Hellenistic era brought sciences to an apogee:

The Archimedean method for the determination of areas and volumes related to the idea of integration developed between the 17^{th} and 19th centuries;

The profound work of Appolonius on conics gaves the idea on which Viéte, Fermat, and Descartes built in 17^{th} century Analytic Geometry.

Hipparchus discovered the precession of equinoxes, which is caused by the cyclic precession of the Earth's axis of rotation.

Claudius Ptolemy applied the theory of epicycles to compile a systematic account of Greek astronomy. His theory generally fitted the data available to him with a good degree of accuracy, and his book, the Almagest molded astronomy for the next millennium and a half.[7]

Pappus had employed an analytic method for the discovery of theorems and the construction of problems; in analysis, by contrast to synthesis, one proceeds from what is sought until one arrives at something known.

Diophantus in his Arithmetica conceives the idea of the unknown quantity, uses abbreviations to express the powers of the unknown and the subtraction (λείψις).[8] These abbreviations are also used in the process of the working out of the equations. His method of solving equations was the start point not only for the Arabs, but also for Viéte and Descartes to formulate Algebra.

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[6] Aristoteles, *Analytica Posteriora*, I. Bekker 77a 1-3 (Thesaurus Linguae Graecae)

[7] *Ancient Middle Eastern and Greek Astronomy*, Britannica CD. Version 97. Encyclopaedia Britannica, Inc., 1997

[8] *Diophantus of Alexandria*, Arithmetica I, pp.1.1- 12.21

After the 3rd A.D. century, the production of new ideas fell in decline. At that time commentaries and explanations of the great works of the past appeared. Possible reasons for this decline could be the discontinuance of the oral tradition and the new philosophical and religious movements.

In the Byzantine era, a few new ideas appeared; its most important contribution in the History of Science was the preservation of ancient scientific heritage mainly by monks who copied, made anthologies and commented the work of ancient Greeks.

In the 7th century, the Arabs, inspired by their new religion, burst out of the Arabian Peninsula and laid the foundations of an Islamic state. For the Arabs, the ancient sciences were precious treasures. Contact with Hindu mathematics and the requirements of astronomy, mainly for religious reasons, stimulated the study of numbers and of geometry.

The writings of the Greeks were, therefore, eagerly sought and translated, and thus much of the science of antiquity passed into the Islamic culture. Greek medicine, Greek astronomy and astrology, and Greek mathematics, together with the great philosophical works of Plato and, particularly, Aristotle were assimilated in Islam by the end of the 9th century.[9]

This fruitful period begins under the Caliph Harun al-Rashid, the fifth Caliph of the Abbasid dynasty, whose reign began in 786 AD. He encouraged scholars, and the first translations of Greek texts into Arabic, such as Euclid's *Elements* by al-Ḥajjāj, were made during al-Rashid's reign. The next Caliph, al-Ma'mūn (813-833 AD), encouraged learning even more strongly than his father al-Rashid, and he set up the House of Wisdom in Baghdad, which became the centre for both the work of translating and of research. Al-Kindi (born 801) and the three Banu Musa brothers worked there, as did the famous translator Hunayn ibn Ishaq.

The translations into Arabic at this time were made by scientists and mathematicians such as those named above, not by language experts ignorant of mathematics, and the need for the translations was stimulated by the most advanced research of the time. It is important to realize that translating was

[9] *Science in Islam*, Britannica CD. Version 97. *Encyclopedia Britannica*, Inc., 1997

not done for its own sake, but was done as part of the current research effort.[10]

The Arabs appropriated not only the knowledge of Greeks, but also the Greek conception of the world and the Greek methodology of the science. As Lindberg says[11], the Islamic science was built on Greek foundations and was formed according to Greek architectural principles; the Muslims did not try to demolish the Greek structure, but they worked for its completion.

Al-Khwarizmī put the foundation of Arabic algebra; Omar Al-Khayyam (b.1048) made a complete classification of cubic equations, solving them by means of intersecting conic sections; Sharaf Al-Din Al-Tusi (b.1135) studied curves by means of equations and inaugurated the beginning of algebraic geometry.

Al-Karajī (b.953) made algebra free of geometrical operations substituting them by arithmetical ones; Al-Samawal (b.1130) following the same tradition declares that Algebra is concerned with operating on unknowns using all the arithmetical tools, in the same way as the arithmetician operates on the unknown.

Al-Birūnī (b.973) held extensive studies on astronomy and geography, using the projection of a hemisphere onto the plane.

Thābit ibn Qurra (b.836), Al-Baghdādī, Al- Haytham worked on Number Theory.

Ibrāhīm ibn Sinān (born 908), who introduced a method of integration more general than that of Archimedes and al-Quhi (born 940) were leading figures in a revival and continuation of Greek higher geometry in the Arab world.

Ibrahim ibn Sinan and his grandfather Thābit ibn Qurra both studied curves required in the construction of sundials.

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[10] O'Connor, J. J. and Robertson, E. F., *Arabic mathematics: forgotten brilliance*. On http:// www-history.mcs.st-andrews.ac.uk

[11] Lindberg, David, *The beginnings of Western Science*, The University of Chicago Press, Chicago, 1992

Nāṣir al-Dīn al-Ṭūsī (born 1201) made the most significant development of Ptolemy's model of the planetary system up to the development of the heliocentric model in the time of Copernicus.[12]

Many of the ideas, which were previously thought to have been brilliant new conceptions due to European mathematicians of the 16th, 17th and 18th centuries, are now known to have been developed by Arab mathematicians around four centuries earlier.

At the western part of Arabic Lands, in Cordoba, Prince ʿAbd al Raḥmān II created in 9th century a great library, where the translated works came directly from Baghdad. A school for Mathematics and Astronomy founded also there by Maslama. The numerous students of that school spread to the kingdoms of Sevilla, Granada, Zaragoza and Toledo. In the monastery of Ripoll, in the province of Gerona, many manuscripts from East and West - among them mathematical ones- were collected. In the 11th century a new phase of mathematics began with the translations from Arabic. In 1126 was founded in Toledo a school for translators, its aim had been the translation of the great Greek and Arabic works into Latin[13]. Along with philosophy, astronomy, astrology, and medicine, important mathematical achievements of the Greek, Indian, and Islamic civilizations became available in the West. Particularly important were Euclid's Elements, the works of Archimedes, and al-Khwarizmī's treatises on arithmetic and algebra. Western texts called algorismus (a Latin form of the name al-Khwarizmī), introduced the Hindu-Arabic numerals and applied them in calculations. Thus modern numerals first came into use in universities and then became common among merchants and other laymen.[14]

The spirit and the wisdom of Greeks met the restless minds of the Renaissance and fired the beginning of a new era in science. Every new idea and theory becomes subject of discussion and is exposed to a critique, which conduces to its improvement.

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[12] See O'Connor and Robertson, *Arabic mathematics: forgotten brilliance*. On http:// www-history.mcs.st-andrews.ac.uk

[13] Veguín Casas, María Victoria, *Mathematicas y Camino de Santiago*, pp.162-165. Ediciones del Orto, Madrid 1998

[14] *The transmission of Greek and Arabic learning*: Britannica CD. Version 97. Encyclopaedia Britannica, Inc., 1997

The scientists Nicolaus Copernicus, Johannes Kepler, and Galileo, following the Platonic ideas of the philosophical importance of mathematics and the Pythagorean attempt to discover the secrets of the heavens in terms of number and exact calculation[15], supported the idea of the heliocentric model -suggested by the Greek astronomer Aristarchus of Samos.

Sustained by a vision of mathematical harmonies in the skies, a vision he derived from the philosophy of Plato and the mathematics of the Pythagoreans, Kepler tried to relate planetary orbits with the five regular solids, or "Platonic bodies": tetrahedron (formed by four equilateral triangles), cube, octahedron (formed by eight equilateral triangles), dodecahedron (12 pentagons), and icosahedron (20 equilateral triangles) in "Mysterium cosmographicum", published in 1596. Later, trying to determine the orbit of Mars -using the observation results of Tycho Brahe- Kepler discovered the three laws describing the motion of the planets, putting out of use the Ptolemaic system. Following the model established by Archimedes, Kepler, in his volumetric researches, investigated the properties of nearly 100 solids of revolution -made by rotating a two-dimensional surface on one of its axes- enormously extending the range of Archimedes' results.[16]

Galileo, following the Greek tradition of proofs, introduced the experiment to study the movement of bodies and used the telescope to affirm the heliocentric system, supported by Copernicus.

In artem analyticem isagoge ("Introduction to the Analytic Arts"; 1591) Viéte, as part of his program of rediscovering the method of analysis used by the ancient Greek mathematicians, proposed new algebraic methods, but he saw this as an advancement over the ancient method, a view he arrived at by comparing the geometric analysis contained in Book VII of Pappus' Collection with the arithmetic analysis of Diophantus' Arithmetica.[17]

Leucippus of Miletus (5th century BC) is thought to have originated the atomic philosophy. Democritus of Abdera (460-370 BC) developed further this philosophy and named the building blocks of matter atomon, meaning "indivisible". According to Democritus, nothing is coming into being

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[15] Humanism, Britannica CD. Version 97. *Encyclopaedia Britannica*, Inc., 1997

[16] Johannes Kepler, Britannica CD. Version 97. *Encyclopaedia Britannica*, Inc., 1997

[17] Mathematics in the 17th and 18th centuries: The 17th century: Analytic Geometry. Britannica CD. Version 97. *Encyclopaedia Britannica*, Inc., 1997

from something not existing and nothing is destructed to something not existing. Atoms are infinite in size and multitude. They move swirling everywhere and they generate everything: fire, water, air, earth, because they are systems of atoms. [18] The atoms move in the void eternally. They are unbreakable, indivisible, unchangeable, and unalterable in quality. The qualities are noticeable because of the combination of atoms.[19] The British chemist and physicist John Dalton converted the atomic philosophy of the Greeks into a scientific theory at the beginning of 19th century.

The earliest philosophers of Greece were theorists of the physical world. The Renaissance continued this breadth of conception characteristic of the Greeks. Galileo and Descartes were mathematicians, physicists, and philosophers at once; and physics retained the name of "natural philosophy" at least until the death of Sir Isaac Newton in 1727.[20]

The multifariousness of today's sciences obliges the scientists to follow very narrow specification. It is rare a philosopher or a historian or a philologist to be also a mathematician or a physicist or a chemist. This makes difficult the spherical view and understanding of the world.

History and Philosophy of science is inseparable part of both, classical studies and exact science. It provides indispensable links with Classical Philosophy, History and the evolution of the exact sciences.

In order to achieve global classical studies, their bounds have to be extended and the study of History and Philosophy of Science must be encouraged.

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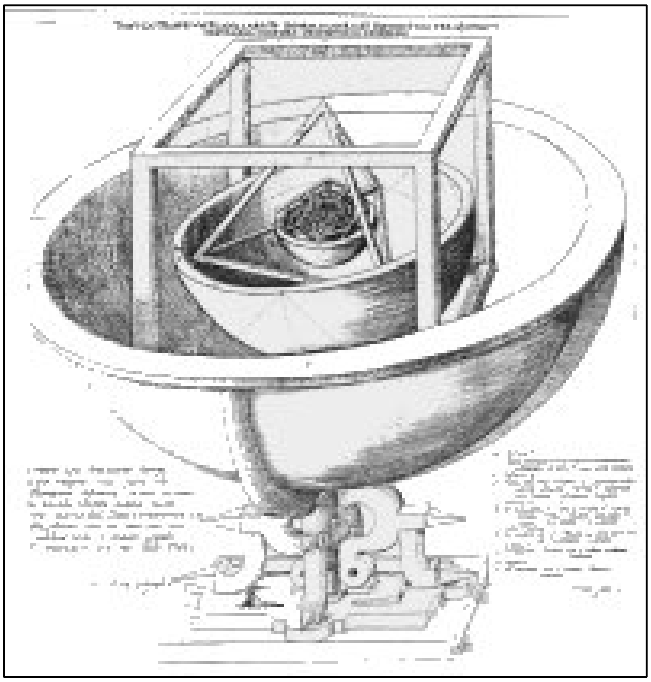
[18] Democritus, *Testimonia*, Fragm. 1 DIOG. IX 34ff. [44]

[19] Democritus,*Testimonia*, Fragm.49 GALEN. de elem. sec. Hipp.I 2 [I 417 K., 3, 20 Helmr.]

[20] Philosophy of nature, Britannica CD. Version 97. *Encyclopaedia Britannica*, Inc., 1997

An early Arabic manuscript from 14th century, which contains the principal work of Nasir ad-Din at-Tusi, "Tadhkira fi ilm al-Haya" (Memoir on Astronomy).

2. For the photo with the Arabs Arabic Observatory



Joh. Kepler, *Mysterium Cosmographicum* Tübingen, 1596. (p. 26 - 27)

3. A figure from Mysterium Cosmographicum, see the possibility of publishing with the text. The comments are in the attachment.